

## How to analyze given algorithm/pseudocode

Suppose we are given some pseudocode using for loops etc.

Step 1 We analyze the pseudocode using the techniques discussed in class (nested for loops, etc).

Using this, we are able to derive a hypothesis on what the time complexity is, asymptotically.

Step 2 Now, we enter the numerical/practical portion where we have to validate our hypothesis.

To validate our hypothesis, we implement the pseudocode in any language like Java/C# etc and find the elapsed time.

Then, we compare the theoretically computed value (the hypothesis) with the numerically calculated elapsed time.

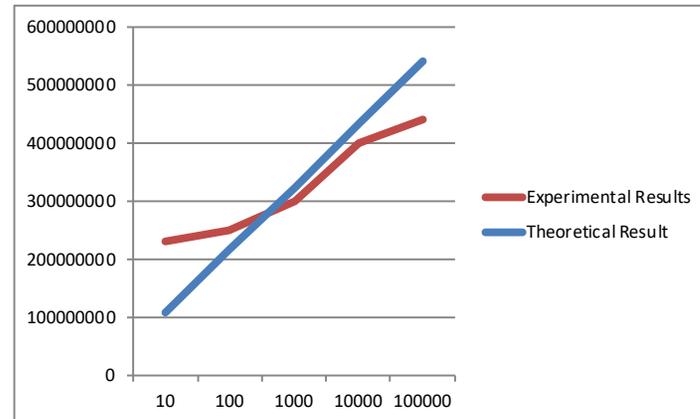
Step 2b However, we have one detail here.

The theoretical values do not have any units, since we only say something like  $O(n^2)$ .

The numerical values have units like millisecond, nanosecond etc.

For this we need to use scaling on one of the values.

| n      | Experimental Result, in ns | Theoretical Result | Scaling Constant | Adjusted Theoretical Result |
|--------|----------------------------|--------------------|------------------|-----------------------------|
| 10     | 230878766                  | 3.32192809         |                  | 108258292                   |
| 100    | 250789567                  | 6.64385619         |                  | 216516584                   |
| 1000   | 300457235                  | 9.96578428         |                  | 324774876                   |
| 10000  | 400895231                  | 13.2877124         |                  | 433033168                   |
| 100000 | 440853582                  | 16.6096405         |                  | 541291460                   |
|        | 324774876.2                | 9.96578428         | 32588993.2       |                             |



Step 3 Now we simply plot the two series (experimental result vs. adjusted theoretical result)

Step 4 Analyze the plots and reach a conclusion.

If the plots are diverging that is a hint that our analysis (and the subsequent hypothesis) may not be correct.

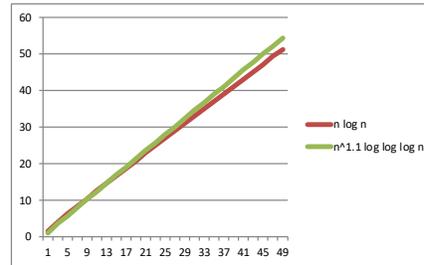
If the experimental result plot is too jumpy, we can try higher n values (computer too fast for small calculations)

## How to compare two Asymptotic Functions Numerically

Suppose we have to compare  $n \log n$  vs.  $n^{1.1} \log \log n$

Some n values.. To compare, we simply calculate numerical values. To Plot, we take a log based 10 of all of these

| n          | n log n     | $n^{1.1} \log \log n$ | $\log(a,10)$ | $\log(b,10)$ | $\log(c,10)$ |
|------------|-------------|-----------------------|--------------|--------------|--------------|
| 10         | 33.21928095 | 9.976433658           | 1            | 1.52139023   | 0.99897532   |
| 1000       | 9965.784285 | 3451.54795            | 3            | 3.99851148   | 3.53801391   |
| 100000     | 1660964.047 | 638567.5564           | 5            | 6.22036023   | 5.80520685   |
| 10000000   | 232534966.6 | 109383809             | 7            | 8.36648827   | 8.03895304   |
| 1000000000 | 29897352854 | 18216762685           | 9            | 10.4756327   | 10.2604712   |
| 1E+11      | 3.65412E+12 | 2.99138E+12           | 11           | 12.5627829   | 12.4758717   |
| 1E+13      | 4.31851E+14 | 4.87164E+14           | 13           | 14.6353336   | 14.6876755   |
| 1E+15      | 4.98289E+16 | 7.8912E+16            | 15           | 16.6974815   | 16.8971431   |
| 1E+17      | 5.64728E+18 | 1.27346E+19           | 17           | 18.7518391   | 19.1049861   |
| 1E+19      | 6.31166E+20 | 2.04947E+21           | 19           | 20.8001438   | 21.3116421   |
| 1E+21      | 6.97605E+22 | 3.29153E+23           | 21           | 22.8436095   | 23.5173976   |
| 1E+23      | 7.64043E+24 | 5.27776E+25           | 23           | 24.8831181   | 25.7224494   |
| 1E+25      | 8.30482E+26 | 8.45158E+27           | 25           | 26.9193302   | 27.9269378   |
| 1E+27      | 8.96921E+28 | 1.35197E+30           | 27           | 28.952754    | 30.1309665   |
| 1E+29      | 9.63359E+30 | 2.16079E+32           | 29           | 30.9837882   | 32.3346134   |
| 1E+31      | 1.0298E+33  | 3.45095E+34           | 31           | 33.0127519   | 34.5379392   |
| 1E+33      | 1.09624E+35 | 5.50797E+36           | 33           | 35.0399042   | 36.7409912   |
| 1E+35      | 1.16267E+37 | 8.78633E+38           | 35           | 37.0654583   | 38.9438077   |
| 1E+37      | 1.22911E+39 | 1.40094E+41           | 37           | 39.089592    | 41.1464194   |
| 1E+39      | 1.29555E+41 | 2.23281E+43           | 39           | 41.1124548   | 43.3488518   |
| 1E+41      | 1.36199E+43 | 3.55734E+45           | 41           | 43.1341741   | 45.5511259   |
| 1E+43      | 1.42843E+45 | 5.66578E+47           | 43           | 45.1548587   | 47.7532593   |
| 1E+45      | 1.49487E+47 | 9.02126E+49           | 45           | 47.1746027   | 49.9552672   |
| 1E+47      | 1.56131E+49 | 1.43603E+52           | 47           | 49.1934881   | 52.1571622   |
| 1E+49      | 1.62774E+51 | 2.28536E+54           | 49           | 51.2115863   | 54.3589553   |



What the curve shows is that while the functions are similar, they are still DIVERGING  
We observe that  $n^{1.1} \log \log n$  is growing faster (albeit ever so slightly) compared to  $n \log n$